Automata with Modulo Counters and Nondeterministic Counter Bounds

Daniel Reidenbach, **Markus L. Schmid**,
Loughborough University UK

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Nondeterministically Bounded Modulo Counter Automata (NBMCA)

- A two-way input head.
- A constant number of k counters.
- A finite state control.

Counters

Each counter consists of

- a counter value,
- a counter bound.

The counter value

- can be incremented or left unchanged,
- starts again at 1 if counter bound is reached.

The counter bound

- changes if a counter is reset.

Transitions

- Current state,
- currently scanned input symbol,
- signals for whether the counters have reached their counter bounds,

$$\Downarrow \text{ (deterministic transition) } \Downarrow$$

- next state,
- input head movement,
- {increment, keep unchanged, reset} each counter.

RESET: A new counter bound is nondeterministically guessed between 0 and $|w|$. Counter value is set to 1.

Repeated Variable Factors

Goal: provide an algorithmic framework for recognising languages that are defined by repeated variable factors (e. g., Pattern languages, extended regular expressions with backreferences):

- $\{x\,x \mid x \in \{\mathtt{a}, \mathtt{b}\}^*\}$,
- $\{x\,\mathtt{a}\,x\,y\,x\,\mathtt{b}\,\mathtt{a}\,y \mid x, y \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^*\}$,

Example

An NBMCA recognising $\{x\,\mathtt{a}\,x\,y\,x\,\mathtt{b}\,\mathtt{a}\,y \mid x, y \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^*\}$:

- We use 2 counters, which are reset initially, i. e., two counter bounds C_1 and C_2 are guessed.
- Check whether $w = u_1\, d_1\, u_2\, u_3\, u_4\, d_2\, d_3\, u_5$, where
  - $|d_1| = |d_2| = |d_3| = 1$,
  - $|u_1| = |u_2| = |u_4| = C_1$,
  - $|u_3| = |u_5| = C_2$.
- Check whether
  - $d_1 = d_3 = \mathtt{a}$,
  - $d_2 = \mathtt{b}$,
  - $u_1 = u_2 = u_4$,
  - $u_3 = u_5$.

Application

NBMCA have been successfully applied in order to identify large classes of pattern languages with a polynomial time membership problem (R., S., CIAA 2010).

Notation

- NBMCA(k) denotes the class of nondeterministically bounded modulo counter automata with k counter.
- 2NFA(k) denotes the class of two-way k-head NFA.
- For a set A of automata, $\mathcal{L}(A)$ describes the set of languages that can be recognised by automata from A.

Expressive Power of NBMCA

Theorem

For every $k \in \mathbb{N}$,

- $\mathcal{L}(\text{NBMCA}(k)) \subseteq \mathcal{L}(\text{2NFA}(2k+1))$,
- $\mathcal{L}(\text{2NFA}(k)) \subseteq \mathcal{L}(\text{NBMCA}(\lceil \frac{k}{2} \rceil + 1))$.

Hierarchy

Theorem

For every $k \in \mathbb{N}$, $\mathcal{L}(\text{NBMCA}(k)) \subset \mathcal{L}(\text{NBMCA}(k+2))$.

Decidability 1/2

Since NBMCA can simulate two-way multi-head automata, the emptiness, infiniteness, universe, equivalence, inclusion and disjointness problem are undecidable for NBMCA.

Definition

(m_1, m_2, l)-REV-NBMCA(k): NBMCA(k) that perform at most m_1 input head reversals, at most m_2 counter reversals and reset every counter at most l times in every accepting computation.

Decidability 2/2

Theorem

The emptiness, infiniteness and disjointness problems are decidable for the class (m_1, m_2, l)-REV-NBMCA.

Decidability 2/2

Theorem

The emptiness, infiniteness and disjointness problems are decidable for the class (m_1, m_2, l)-REV-NBMCA.

Theorem

The emptiness, infiniteness, universe, equivalence, inclusion and disjointness problems are undecidable for the class $(3, \infty, 1)$-REV-NBMCA(3).

Stateless NBMCA

SL-NBMCA(k) denote *stateless* NBMCA(k).

Theorem

For every $M \in$ NBMCA(k), $k \in \mathbb{N}$, with a set of states Q, there exists an $M' \in$ SL-NBMCA($k + \lceil \log(|Q| + 1) \rceil + 2$) with $L(M) = L(M')$.

SL-NBMCA with Restricted Nondeterminism

$1\text{SL-NBMCA}_k(1)$ are *one-way* SL-NBMCA(1), where, for every counter, only the first k resets have an effect (i. e., a new counter bound is guessed).

Theorem

For every $k \in \mathbb{N}$, there exists a language $L \in \mathcal{L}(1\text{SL-NBMCA}_k(1))$ with $L \notin \mathcal{L}(1\text{SL-NBMCA}_{k'}(1))$ for every $k' \in \mathbb{N}$, $k' < k$.

Theorem

There exist infinitely many $k \in \mathbb{N}$ such that $\mathcal{L}(1\text{SL-NBMCA}_k(1))$ and $\mathcal{L}(1\text{SL-NBMCA}_{k+1}(1))$ are incomparable.

Questions

Thank you for your attention.