A Polynomial Time Match Test for Large Classes of Extended Regular Expressions

Daniel Reidenbach, **Markus L. Schmid**,
Loughborough University UK

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Outline

Extended regular expressions (REGEX)

- A mechanism to define formal languages.

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- Practical application.

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- Our model: pattern languages.

Pattern languages 1/3

- Σ: Finite alphabet of terminal symbols.
  (e. g. $\Sigma = \{a, b, c, d\}$)
- X: Infinite alphabet of variables.
  ($X = \{x_1, x_2, x_3, \ldots\}$)
- A string $\alpha \in (\Sigma \cup X)^+$ is called a *pattern*.

Pattern languages 2/3

- *Morphism*: Mapping $\sigma : \Gamma_1^* \to \Gamma_2^*$ with $\sigma(x \cdot y) = \sigma(x) \cdot \sigma(y)$.
- *Substitution*: Morphism $\sigma : (\Sigma \cup X)^* \to \Sigma^*$ with $\sigma(a) = a$ for all $a \in \Sigma$.
- $L_\Sigma(\alpha)$: $\{\sigma(\alpha) \mid \sigma$ is a substitution$\}$.

Pattern languages 3/3

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We will concentrate on patterns in X^+.

Notation

- $\mathrm{var}(\alpha)$: Set of variables occurring in α.
  E. g. $\mathrm{var}(x_1 ab x_2 ba x_1 x_2 c x_3) = \{x_1, x_2, x_3\}$.

- $|\alpha|_{x_i}$: Number of occurrences of variable x_i in α.

Match test for pattern languages

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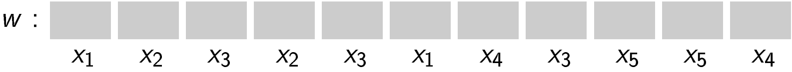

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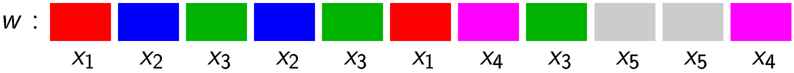

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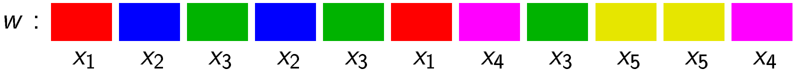

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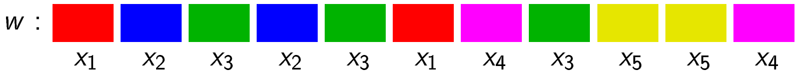

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Theorem

The match test for pattern languages is NP-complete (Angluin, 80).

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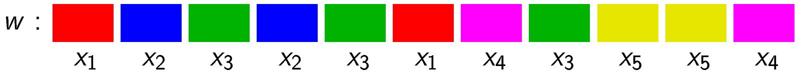

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Question: Are there classes of pattern languages with a polynomial match test?

Simple restrictions

Match test is polynomial if input (α, w) is restricted to

1. at most k different variables (for a constant k),
2. only one occurrence per variable,
3. $|\Sigma| = 1$.

A more useful restriction

We aim for a restriction with

- arbitrarily many variables,
- arbitrarily many occurrences of each variable,
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Main idea: Establish a reasonable automata model to recognize pattern languages.

The Janus automaton

- Two two-way input heads.
- A constant number of k counters.
- A finite state control.

Counters

Each counter consists of

- a counter value,
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The counter bound

- changes if a counter is reset.

Transitions

A transition, depending

- on the current state,
- on the currently scanned input symbols,
- on whether the counters have reached their counter bounds or not,

determines (completely deterministically)

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RESET: A new counter bound is nondeterministically guessed between 0 and $|w|$. Counter value is set to 1.

Notation

- The automata model is denoted by $JFA(k)$,
- $L(M)$ is the language accepted by a $JFA(k)$ M,
- $\mathfrak{L}_k := \{L(M) \mid \text{ M is a } JFA(k)\}$.

Back to pattern languages

Proposition

Let α be a pattern with $|\mathrm{var}(\alpha)| = m$. Then $L_{\Sigma}(\alpha) \in \mathfrak{L}_m$.

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Example:

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Reset all counters and obtain counter bounds C_1, C_2, C_3, C_4, C_5.

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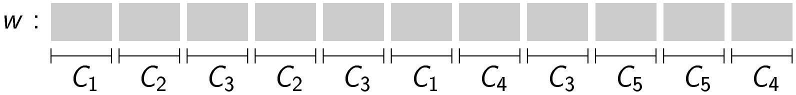

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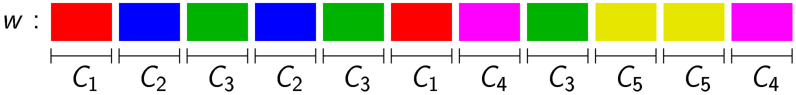

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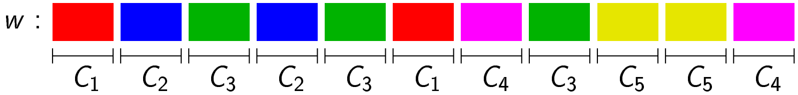

Question

Is it possible to recognise $L_\Sigma(\alpha)$ by a $JFA(k)$ with $k < |\mathrm{var}(\alpha)|$?

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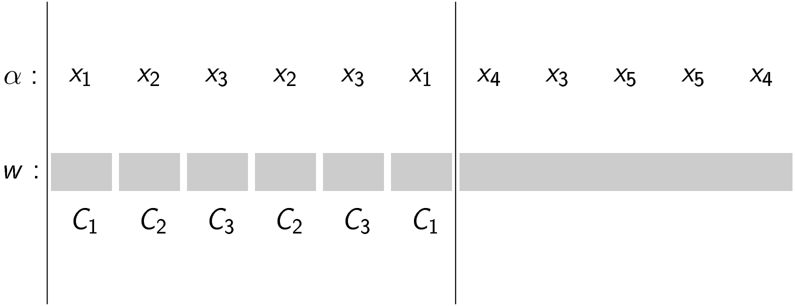

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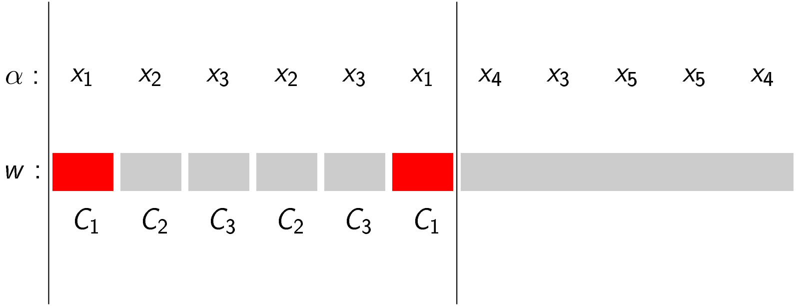

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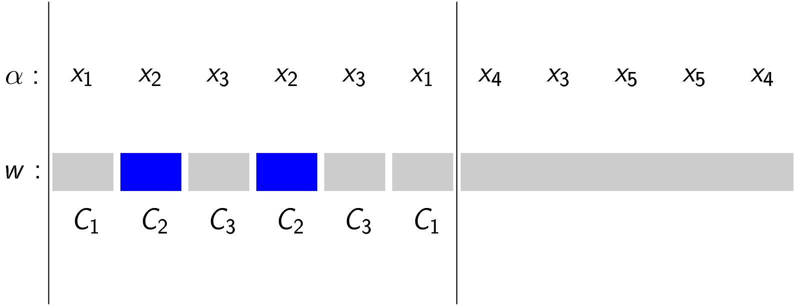

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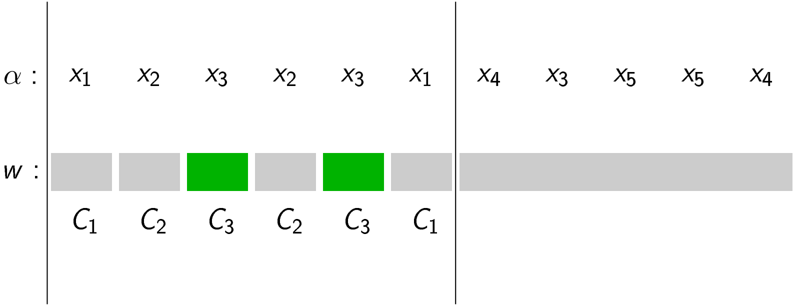

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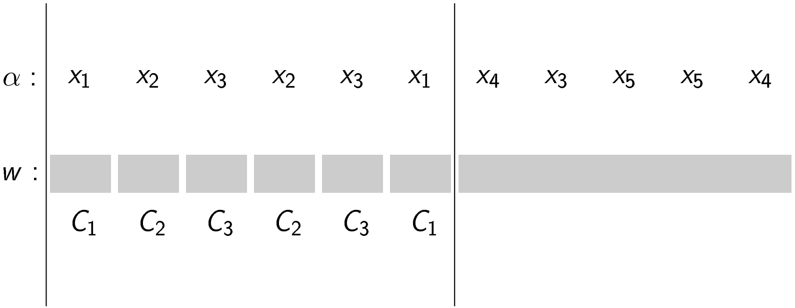

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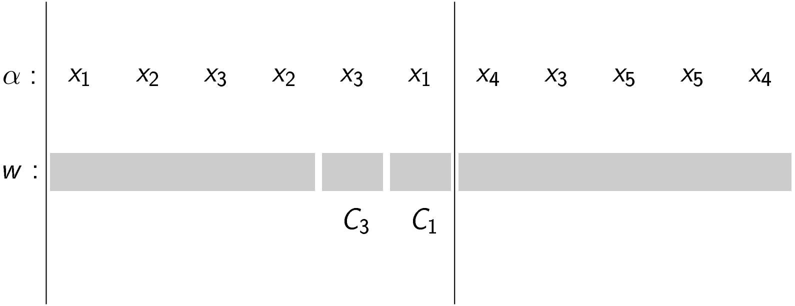

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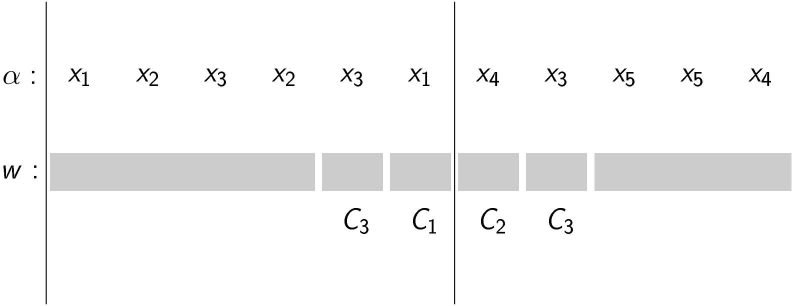

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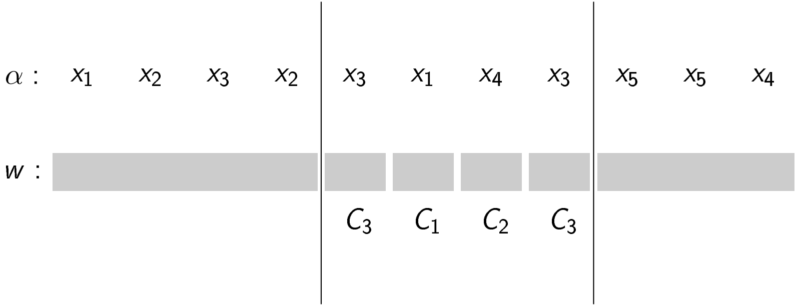

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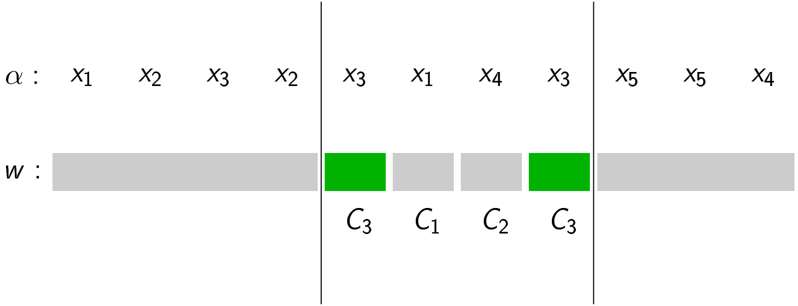

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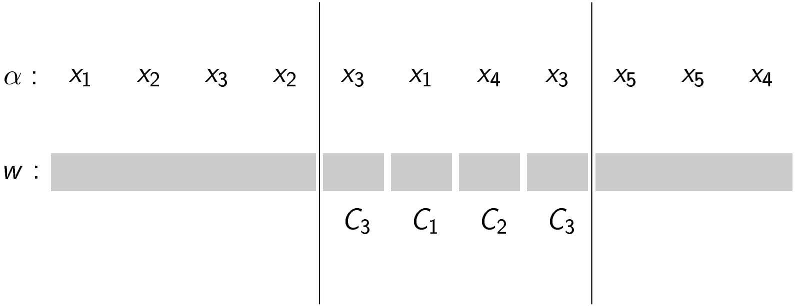

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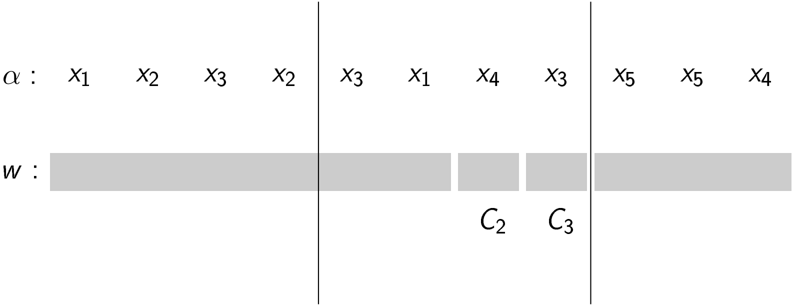

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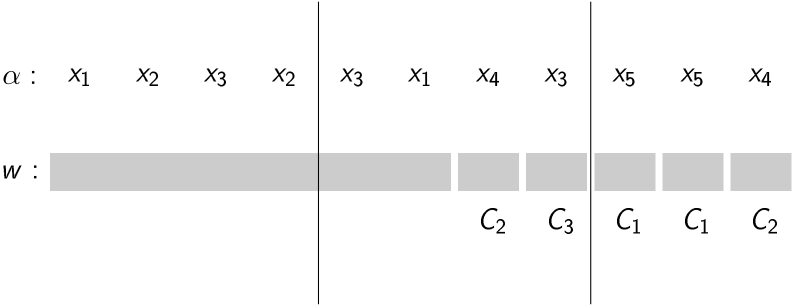

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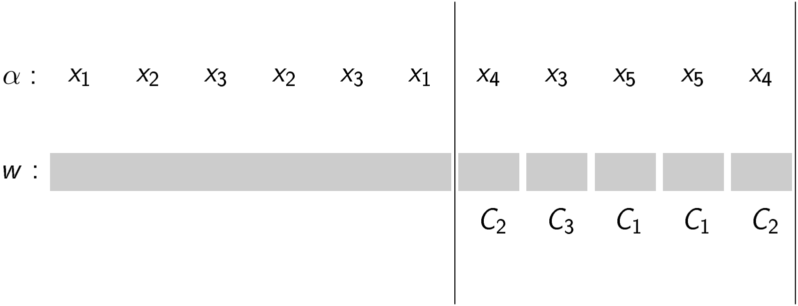

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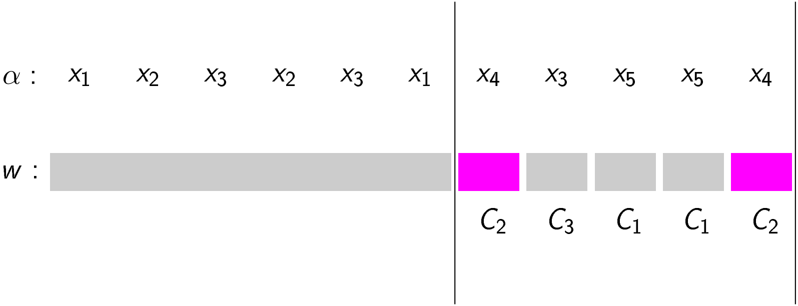

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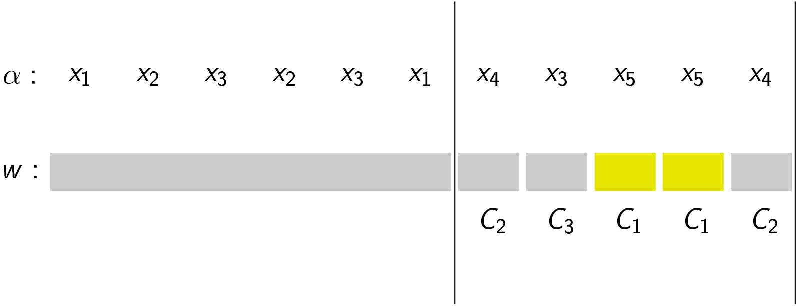

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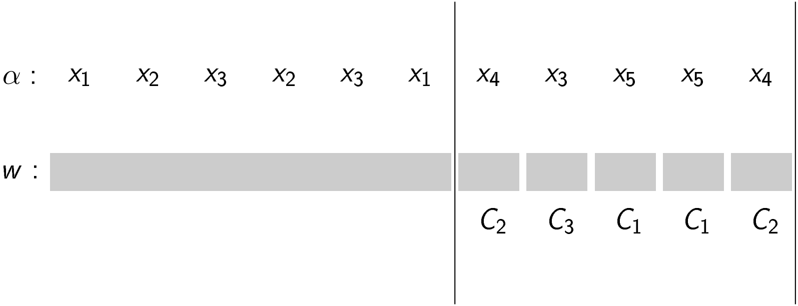

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implies $|\text{var}(\gamma)| \leq k$.

Example

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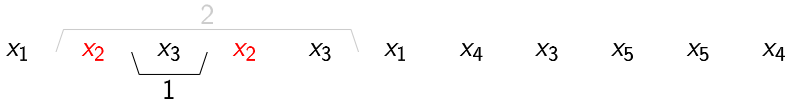

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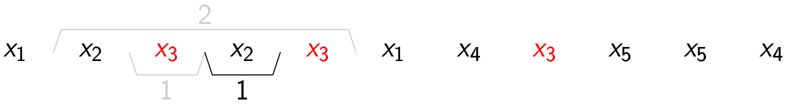

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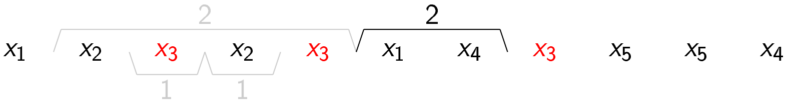

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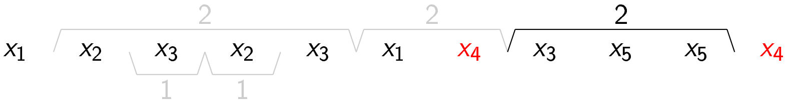

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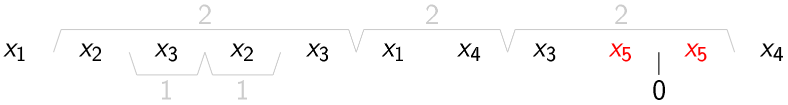
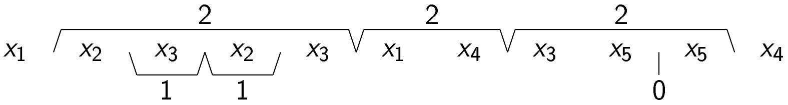


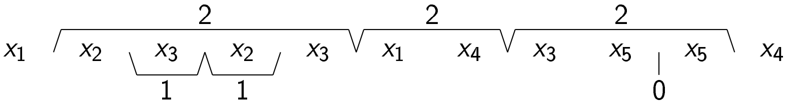
Example

$$\mathrm{vd}(x_1 \cdot x_2 \cdot x_3 \cdot x_2 \cdot x_3 \cdot x_1 \cdot x_4 \cdot x_3 \cdot x_5 \cdot x_5 \cdot x_4) = ?$$



Example

$$\text{vd}(x_1 \cdot x_2 \cdot x_3 \cdot x_2 \cdot x_3 \cdot x_1 \cdot x_4 \cdot x_3 \cdot x_5 \cdot x_5 \cdot x_4) = 2$$



Main Result

Theorem

Let α be a pattern with $\mathsf{vd}(\alpha) \leq k$. Then $L_\Sigma(\alpha) \in \mathfrak{L}_{k+1}$.

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Corollary

The match test for the class $\{L_\Sigma(\alpha) \mid \mathsf{vd}(\alpha) \leq k\}$ is solvable in time $O(|\alpha|^3 \, |w|^{(\mathsf{vd}(\alpha)+4)})$.

Test Results 1/3

Two algorithms:

Test Results 1/3

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- JANUS: Java implementation of our JFA approach for the match test of pattern languages.

Test Results 1/3

Two algorithms:

- JANUS: Java implementation of our JFA approach for the match test of pattern languages.
- JREG: Java regex engine, implemented in the Java core packages.

Test Results 2/3

Test settings:

- $|\mathsf{var}(\alpha)| \in \{10, 15\}$,

Test Results 2/3

Test settings:

- $|\text{var}(\alpha)| \in \{10, 15\}$,
- $\text{vd}(\alpha) \in \{2, 3, 4\}$,

Test Results 2/3

Test settings:

- $|\text{var}(\alpha)| \in \{10, 15\}$,
- $\text{vd}(\alpha) \in \{2, 3, 4\}$,
- For each $x \in \text{var}(\alpha)$, $20 \leq |\alpha|_x \leq 25$,

Test Results 2/3

Test settings:

- $|\mathrm{var}(\alpha)| \in \{10, 15\}$,
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- For each $x \in \mathrm{var}(\alpha)$, $20 \leq |\alpha|_x \leq 25$,
- $w \in \Sigma^+$ with $|\Sigma| = 2$,

Test Results 2/3

Test settings:

- $|\mathrm{var}(\alpha)| \in \{10, 15\}$,
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- For each $x \in \mathrm{var}(\alpha)$, $20 \leq |\alpha|_x \leq 25$,
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- on average $|w| \approx 3\,|\alpha|$,

Test Results 2/3

Test settings:

- $|\text{var}(\alpha)| \in \{10, 15\}$,
- $\text{vd}(\alpha) \in \{2, 3, 4\}$,
- For each $x \in \text{var}(\alpha)$, $20 \leq |\alpha|_x \leq 25$,
- $w \in \Sigma^+$ with $|\Sigma| = 2$,
- on average $|w| \approx 3\,|\alpha|$,
- For each scenario, 250 instances with $w \in L_\Sigma(\alpha)$ and 250 instances with $w \notin L_\Sigma(\alpha)$.

Test Results 3/3

$\lvert var(\alpha) \rvert$	vd(α)	JANUS	JREG	JREG / JANUS
10	2	114 (0)	2653 (0)	23.2
10	3	5272 (0)	234903 (142)	44.56
10	4	36951 (0)	429527 (313)	11.62
15	2	440 (0)	10818 (0)	24.61
15	3	23506 (5)	371430 (232)	15.8
15	4	264239 (119)	526005 (403)	1.99

Questions

Thank you for your attention.

Questions?